

# Wall Following

ROB 102: Introduction to AI & Programming

Lab Session 3

2021/09/24

# Administrative

When batteries are charging, they should be switched to **ON** (—).

When batteries are stored, they should be switched to **OFF** (◦).

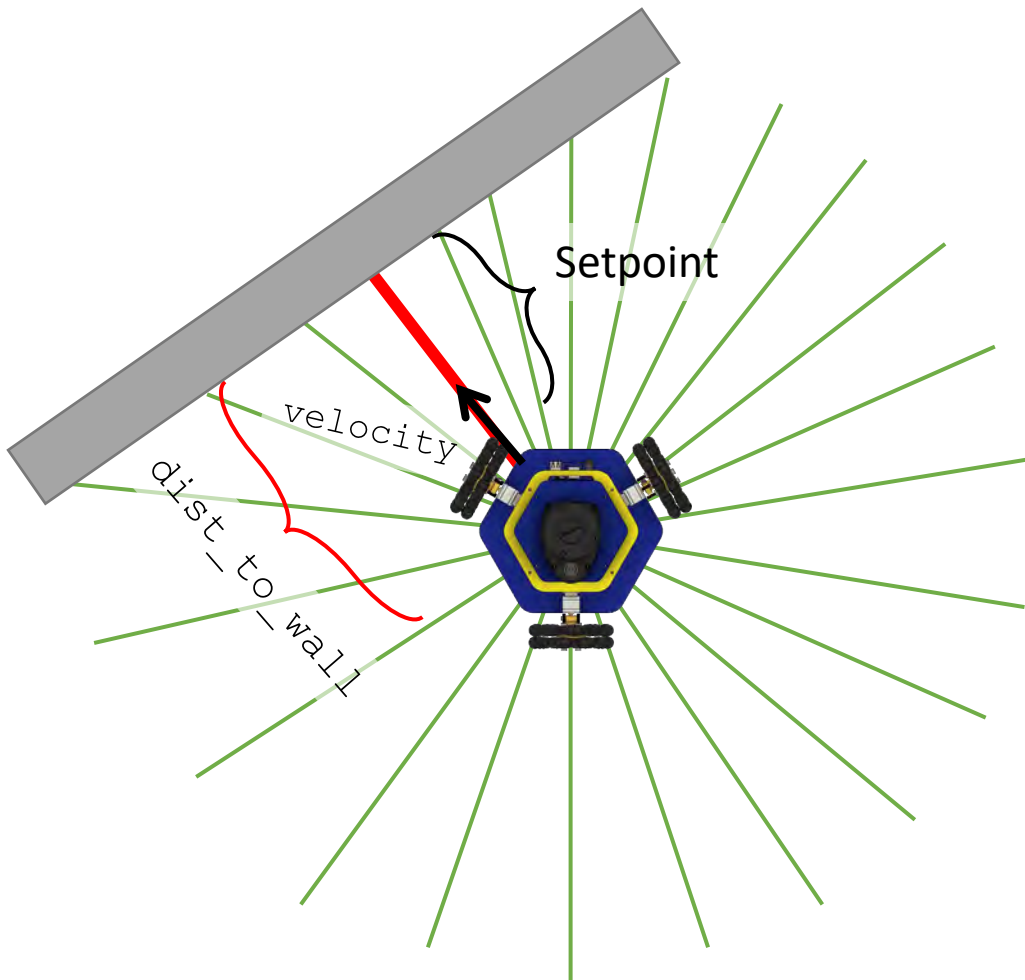
Monday's lecture will be a **C++ review (+ Practice Quiz)**

**Checkpoint:** Part 1 (Drive Square) and Part 2 (Safe Drive) should be completed this week.

# Today...

1. 2D velocity control
2. Driving parallel to a wall (the cross product)
3. Maintaining a distance from the wall
4. Lab time: work on Project 1

# Recall: Bang-Bang Control to Nearest Wall



We can use the same controller (bang-bang or P-control) as last week. But this time, we'll **drive in the direction of the shortest ray.**

We need to:

1. Find the **direction** and **length** of the shortest ray, ✓ **Covered on Wednesday**
2. Drive the robot in any direction. **TODO**

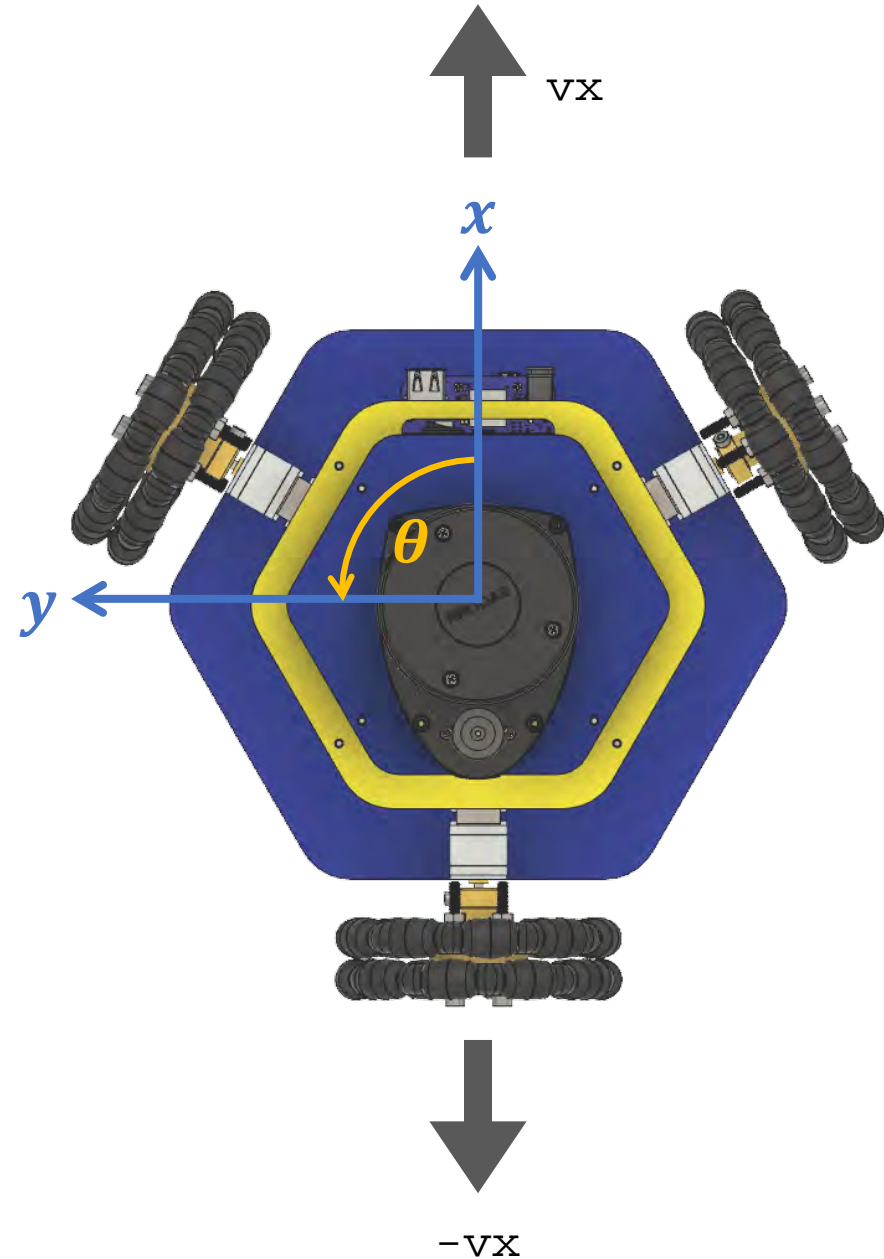
# 2D Velocity Control

Moving the robot forward:

```
drive(vx, 0, 0);
```

Moving the robot backward:

```
drive(-vx, 0, 0);
```



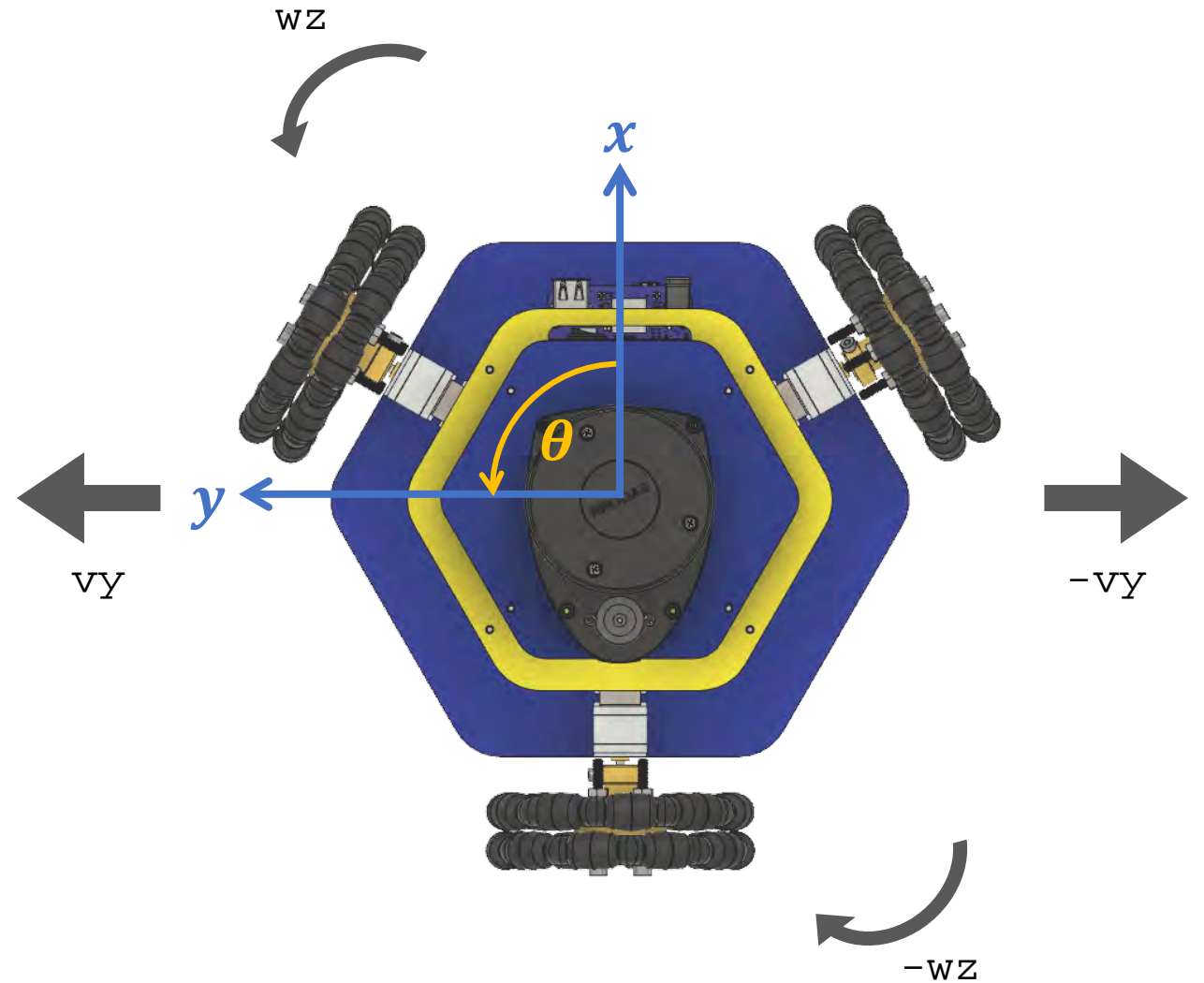
# 2D Velocity Control

Moving the robot left:

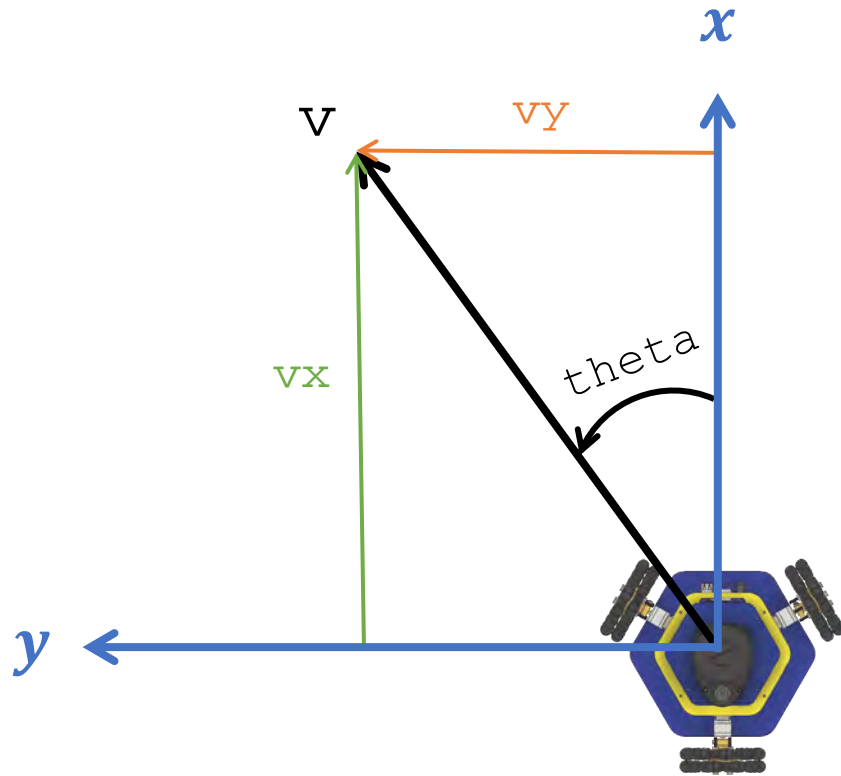
```
drive(0, vy, 0);
```

Rotating counterclockwise:

```
drive(0, 0, wz);
```



# 2D Velocity Control: Trigonometry Review



Our velocity is a 2D vector. There are two ways to represent it:

1. Using the magnitude ( $v$ ) and angle ( $\theta$ )
  - Recall from in-class activity: These are polar coordinates.
2. Using the  $x$  and  $y$  components of the vector ( $v_x, v_y$ )

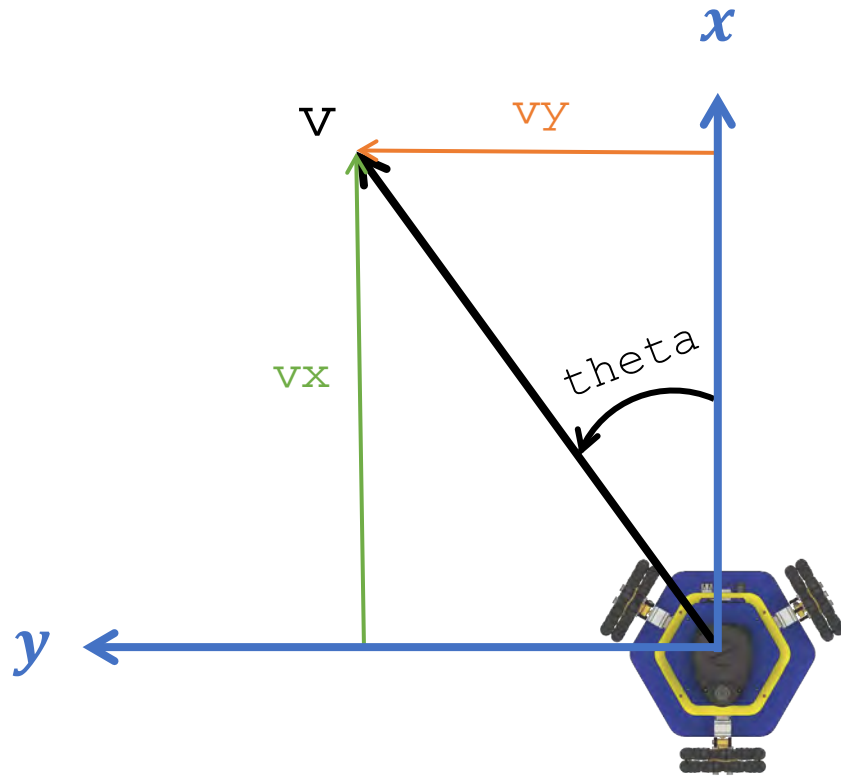
# 2D Velocity Control: Trigonometry Review

To move at velocity  $v$ , at angle  $\theta$  (no angular velocity):

$$v_x = v * \cos(\theta)$$

$$v_y = v * \sin(\theta)$$

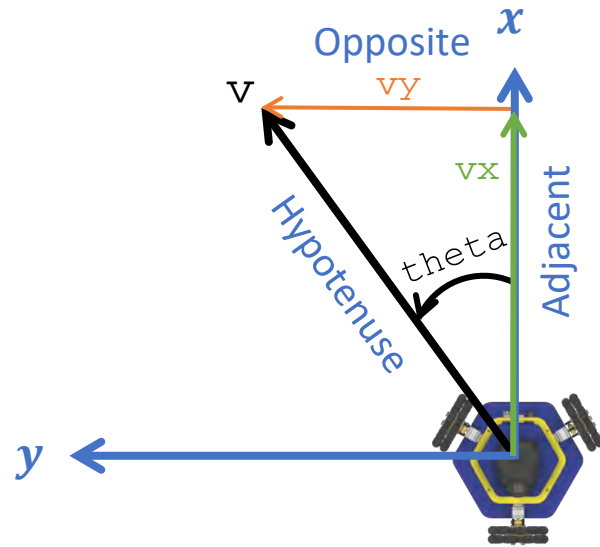
```
drive(vx, vy, 0)
```



This will work for any velocity and angle (try it yourself!)



# Recall: Trigonometry



To move at velocity  $v$ , at angle  $\theta$  (no angular velocity):

$$v_x = v * \cos(\theta)$$

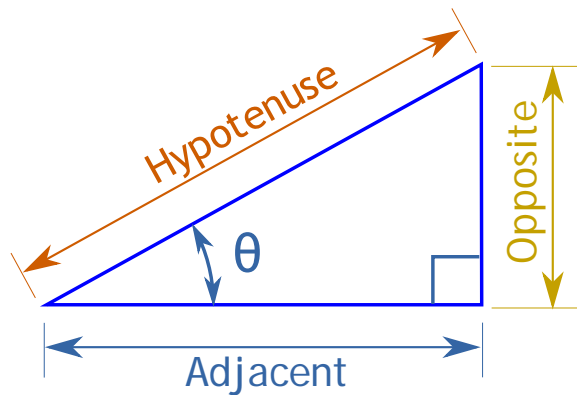
$$v_y = v * \sin(\theta)$$

```
drive(vx, vy, 0)
```

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$



This will work for any velocity and angle (try it yourself!)

# The <cmath> library

```
main.cpp x
1  #include <iostream>
2  #include <cmath> ← Remember to
3
4  int main() {
5      // Common math expressions
6      std::cout << "pow(3, 3) = " << pow(3, 3) << "\n";
7      std::cout << "sqrt(2) = " << sqrt(2) << "\n";
8      std::cout << "abs(-3) = " << abs(-3) << "\n";
9      std::cout << "fabs(-1.5) = " << fabs(-1.5) << "\n";
10     std::cout << "Careful! abs(-1.5) = " << abs(-1.5) << "\n";
11
12     // Trig functions
13     float pi = 3.14159265359;
14     std::cout << "\nsin(1.0) = " << sin(1.0) << "\n";
15     std::cout << "tan(pi) = " << tan(pi) << "\n";
16
17     // Logs & Exponentials
18     float e = 2.71828;
19     std::cout << "\nlog(e) = " << log(e) << "\n";
20     std::cout << "exp(1.0) = " << exp(1.0) << "\n";
21 }
22
```

Contains common math operations.

```
Console Shell
~/Test$ g++ main.cpp -o main
~/Test$ ./main
pow(3, 3) = 27 ← 33
sqrt(2) = 1.41421 ← √2
abs(-3) = 3
fabs(-1.5) = 1.5 ← |-1.5|
Careful! abs(-1.5) = 1

sin(1.0) = 0.841471
tan(pi) = 8.74228e-08

log(e) = 0.999999
exp(1.0) = 2.71828 ← e1.0
~/Test$
```

# The `<cmath>` library

For a list of all the functions:

<https://www.cplusplus.com/reference/cmath/>

This website is a great reference for all things C++!

## *fx* Functions

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### Trigonometric functions

<code>cos</code>	Compute cosine (function )
<code>sin</code>	Compute sine (function )
<code>tan</code>	Compute tangent (function )
<code>acos</code>	Compute arc cosine (function )
<code>asin</code>	Compute arc sine (function )
<code>atan</code>	Compute arc tangent (function )
<code>atan2</code>	Compute arc tangent with two parameters (function )

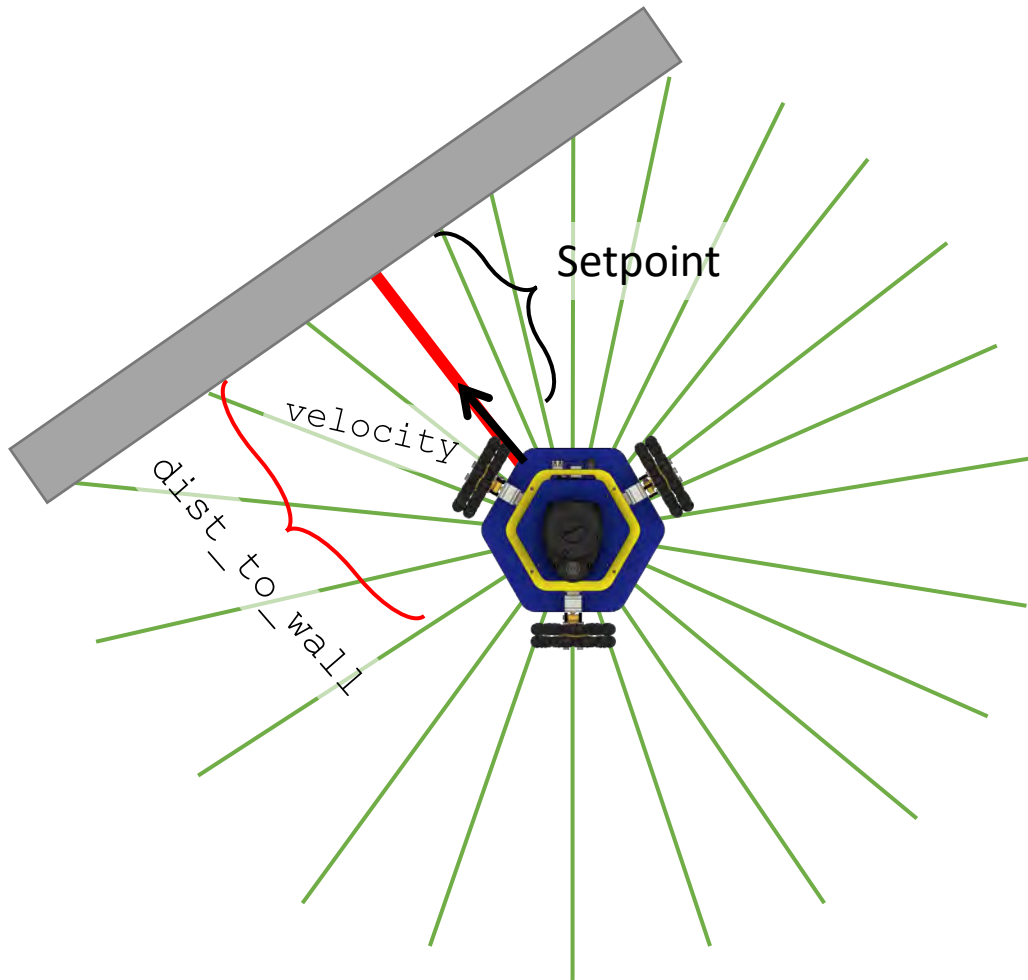
### Hyperbolic functions

<code>cosh</code>	Compute hyperbolic cosine (function )
<code>sinh</code>	Compute hyperbolic sine (function )
<code>tanh</code>	Compute hyperbolic tangent (function )
<code>acosh</code> <small>C++11</small>	Compute area hyperbolic cosine (function )
<code>asinh</code> <small>C++11</small>	Compute area hyperbolic sine (function )
<code>atanh</code> <small>C++11</small>	Compute area hyperbolic tangent (function )

### Exponential and logarithmic functions

<code>exp</code>	Compute exponential function (function )
<code>frexp</code>	Get significand and exponent (function )
<code>ldexp</code>	Generate value from significand and exponent (function )
<code>log</code>	Compute natural logarithm (function )
<code>log10</code>	Compute common logarithm (function )
<code>modf</code>	Break into fractional and integral parts (function )
<code>exp2</code> <small>C++11</small>	Compute binary exponential function (function )
<code>expm1</code> <small>C++11</small>	Compute exponential minus one (function )
<code>ilogb</code> <small>C++11</small>	Integer binary logarithm (function )
<code>log1p</code> <small>C++11</small>	Compute logarithm plus one (function )
<code>log2</code> <small>C++11</small>	Compute binary logarithm (function )
<code>logb</code> <small>C++11</small>	Compute floating-point base logarithm (function )
<code>scalbn</code> <small>C++11</small>	Scale significand using floating-point base exponent (function )
<code>scalbln</code> <small>C++11</small>	Scale significand using floating-point base exponent (long) (function )

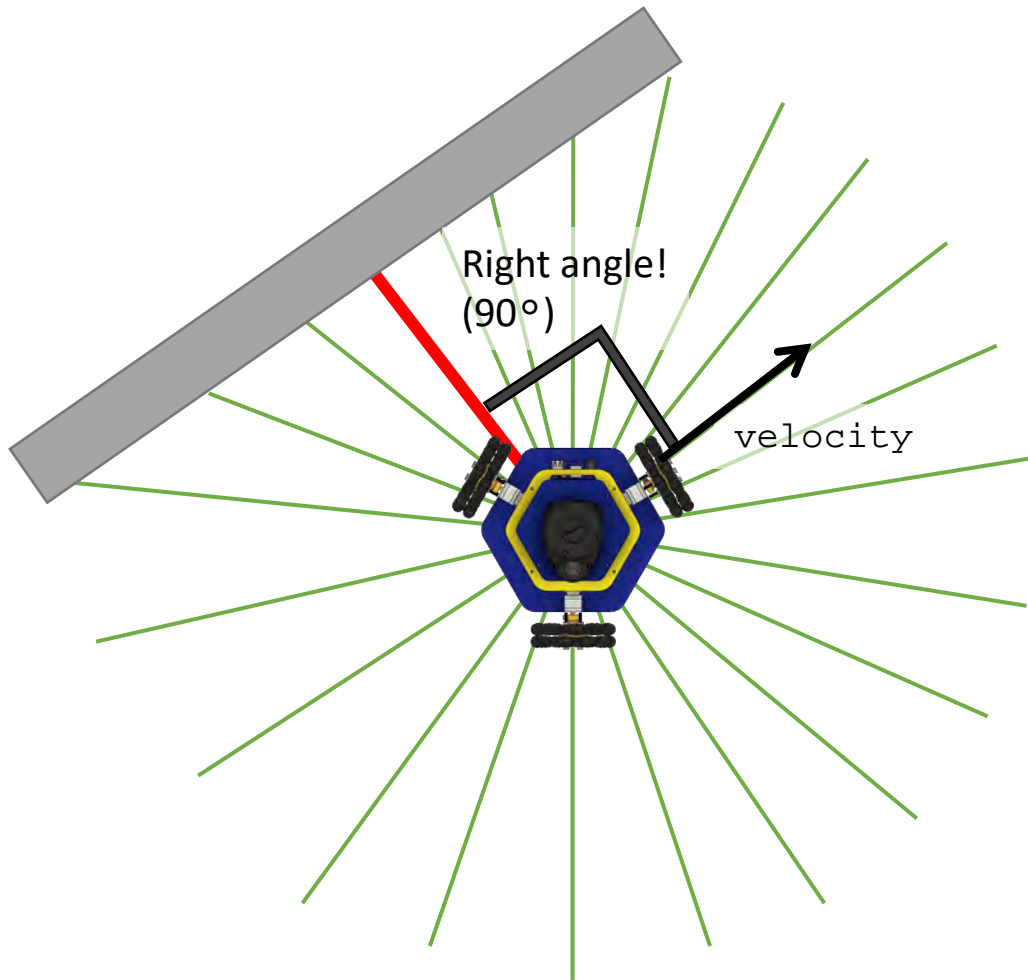
# Wall Following



In class, we discussed how we can maintain a distance to the wall by driving **in the direction of the shortest ray**.

**How can we drive along the wall?**

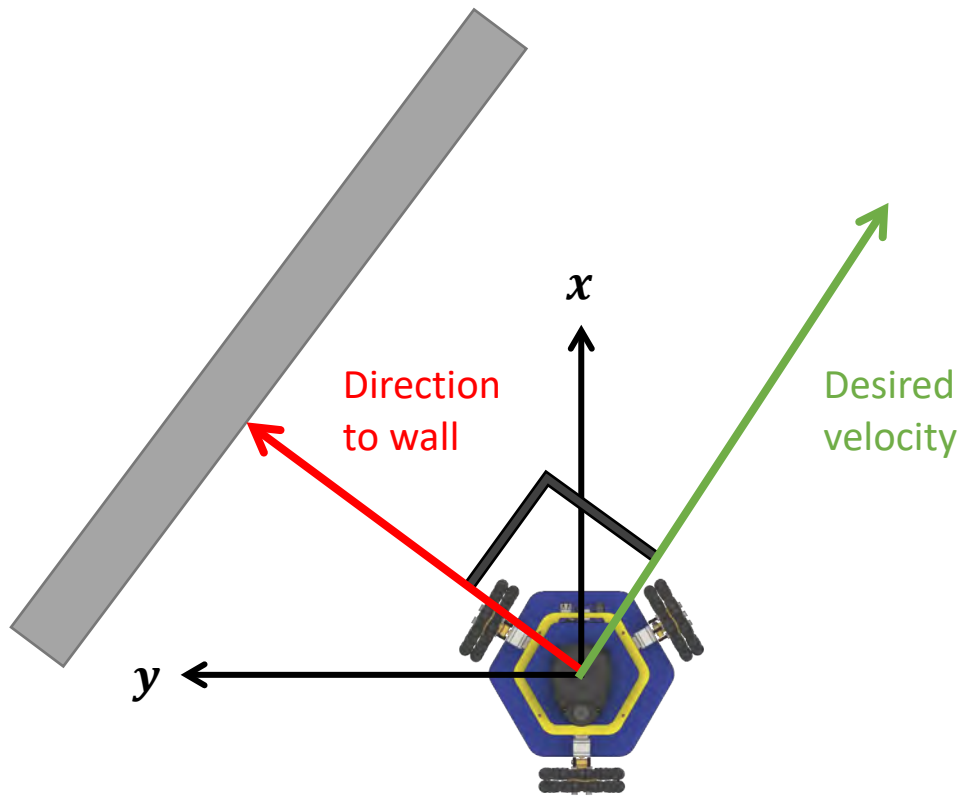
# Wall Following



We can pick a velocity *perpendicular* (at a right angle) to the shortest ray.

How do we calculate this velocity vector?

# Shortest Lidar Ray



We know the magnitude and the angle of the vector pointing to the wall:

```
// Get the distance to the wall.  
float min_idx = findMinDist(scan);  
float dist_to_wall = scan.ranges[min_idx];  
float angle_to_wall = scan.thetas[min_idx];
```

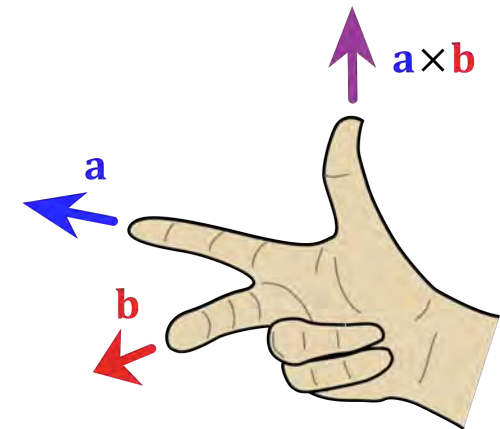
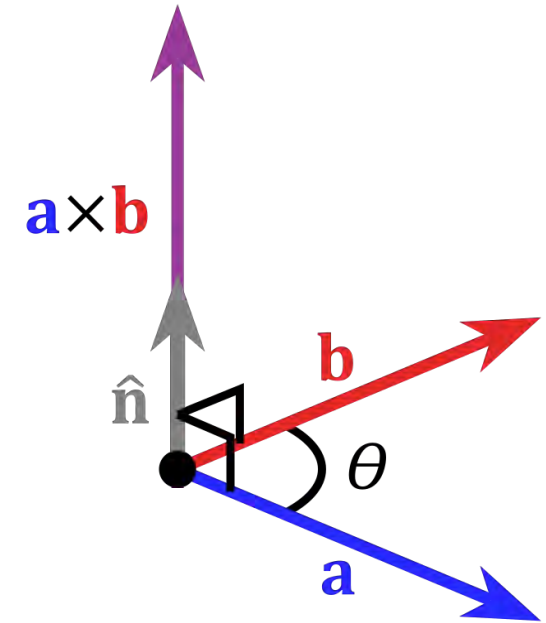
Given the index of the minimum length ray, the magnitude is `dist_to_wall` and the angle is `angle_to_wall`.

# The Cross Product

The **cross product** is an operation that finds a 3D vector *perpendicular* to two other 3D vectors.

The Right-Hand Rule gives the direction of the resulting vector.

**Note:** Our robot can only move in 2D, but the velocity vectors can be written as 3D vectors by setting the z component to zero.



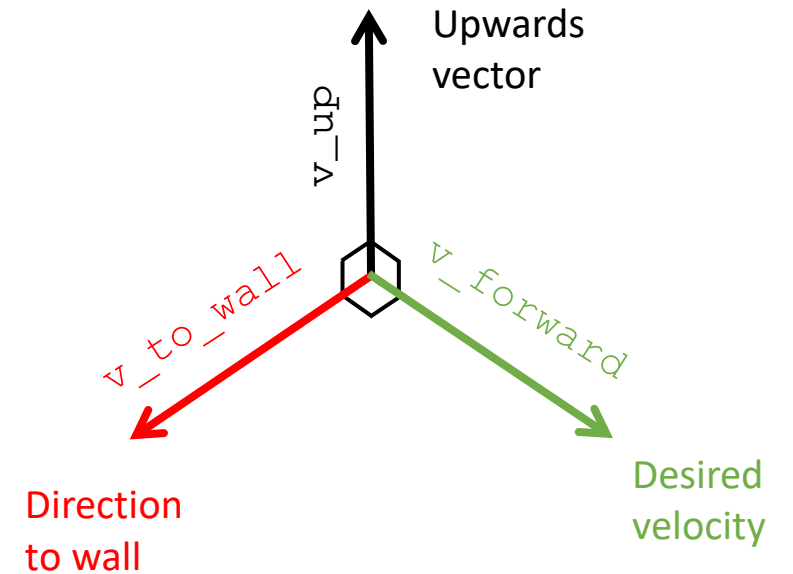
# The Cross Product

The **cross product** is an operation that finds a 3D vector *perpendicular* to two other 3D vectors.

We pick a vector pointing **up** as the second vector:

$$v\_up = [0, 0, 1] \quad // \quad x, y, z$$

It is guaranteed to be perpendicular to the desired velocity.





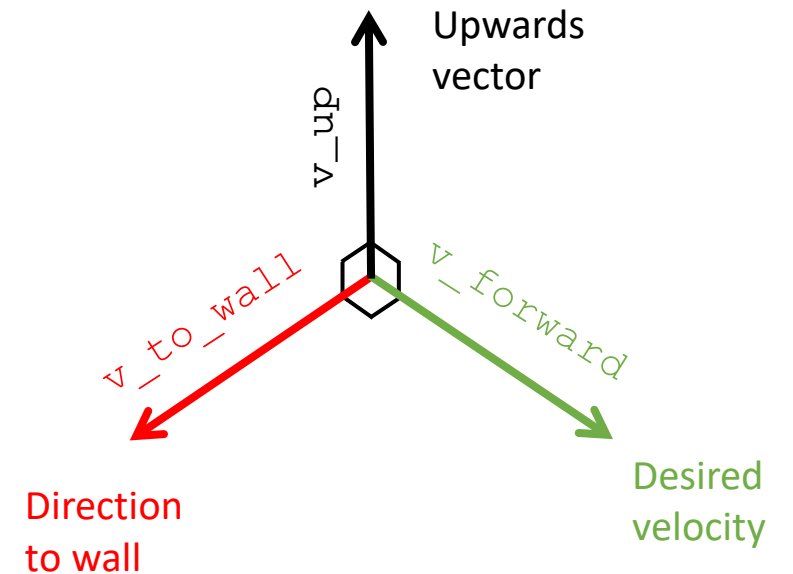
# The Cross Product

We pick a vector pointing **up** as the second vector:

```
v_up = [0, 0, 1] // x,y,z
```

Now we can use the cross product to find the forward velocity vector:

```
v_forward = crossProduct(v_to_wall, v_up)
```

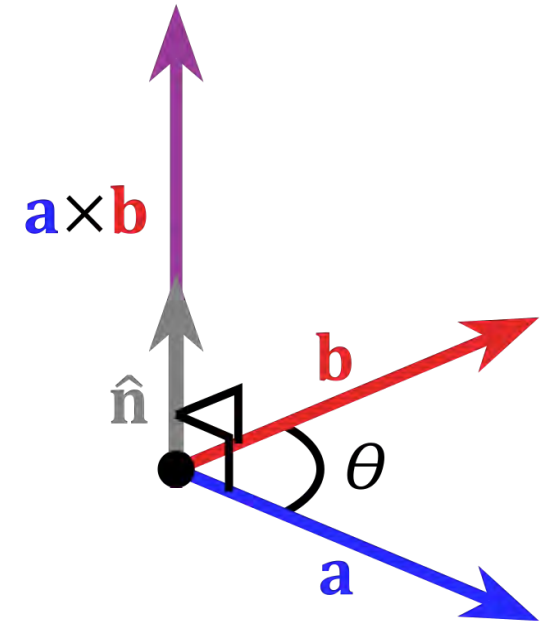


# Computing the Cross Product

Given two vectors  $\mathbf{a} = (a_x, a_y, a_z)$  and  $\mathbf{b} = (b_x, b_y, b_z)$ , their cross product  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$  can be computed like this:

$$\begin{aligned}c_x &= a_y b_z - a_z b_y \\c_y &= a_z b_x - a_x b_z \\c_z &= a_x b_y - a_y b_x\end{aligned}$$

The *magnitude* of  $\mathbf{c}$  will be the area of the parallelogram made by  $\mathbf{a}$  and  $\mathbf{b}$ .



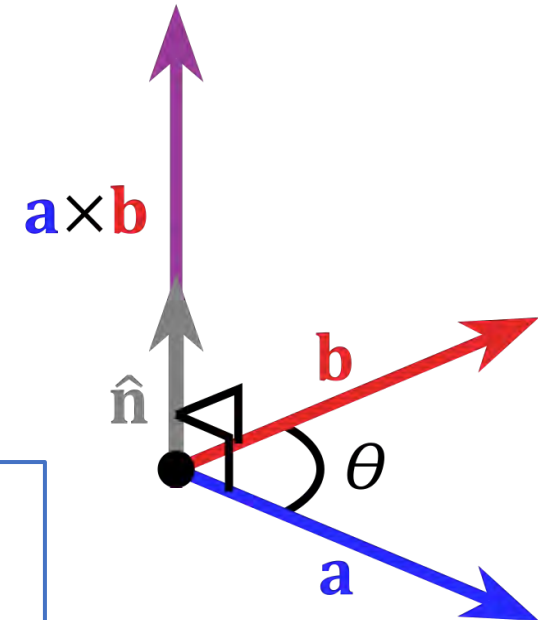
# Computing the Cross Product

Computing  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ :

$$c_x = a_y b_z - a_z b_y$$

$$c_y = a_z b_x - a_x b_z$$

$$c_z = a_x b_y - a_y b_x$$



```
3  Vector3D crossProduct(const Vector3D& v1, const Vector3D& v2)
4  {
5      Vector3D res;
6      /**
7       * TODO: (P1.3.1) Take the cross product between v1 and v2 and store the
8       * result in res.
9       */
10     return res;
11 }
12
```

**Your turn!** Compute the cross product in this function.

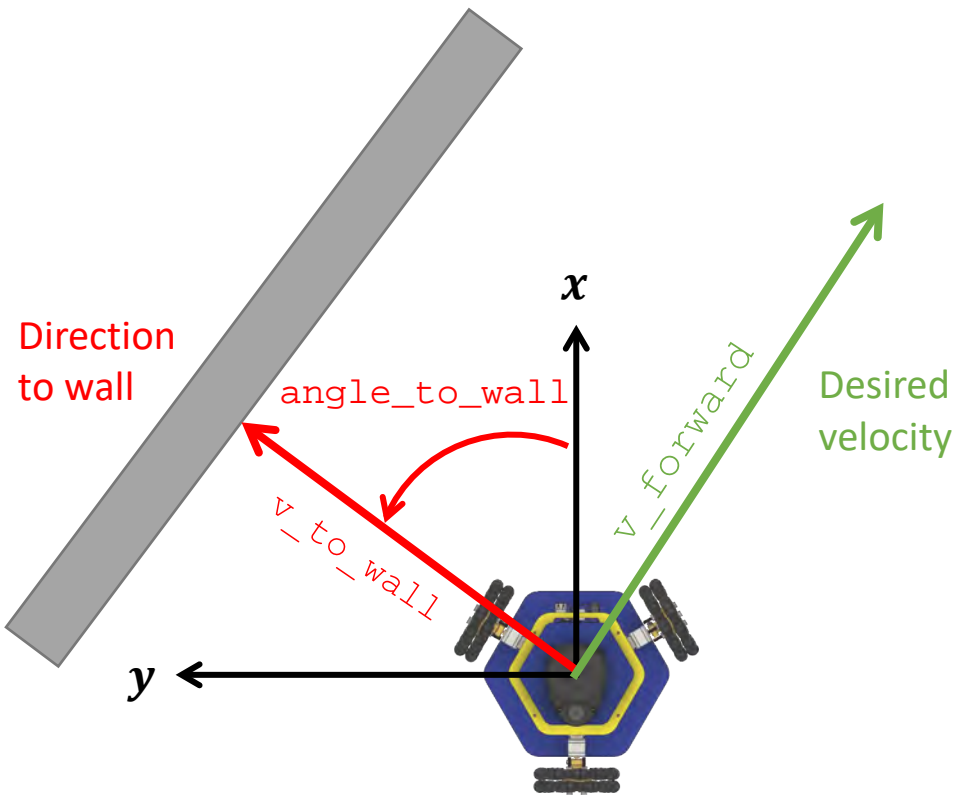
```
25  struct Vector3D
26  {
27      Vector3D() :
28          x(0),
29          y(0),
30          z(0)
31      {};
32
33      float x, y, z;
34  };
35
```

src/common/utils.cpp

include/wall\_follower/common/utils.h

# Computing the Cross Product

We know the magnitude and the angle of the vector pointing to the wall:



```
// Get the distance to the wall.  
float min_idx = findMinDist(scan);  
float dist_to_wall = scan.ranges[min_idx];  
float angle_to_wall = scan.thetas[min_idx];
```

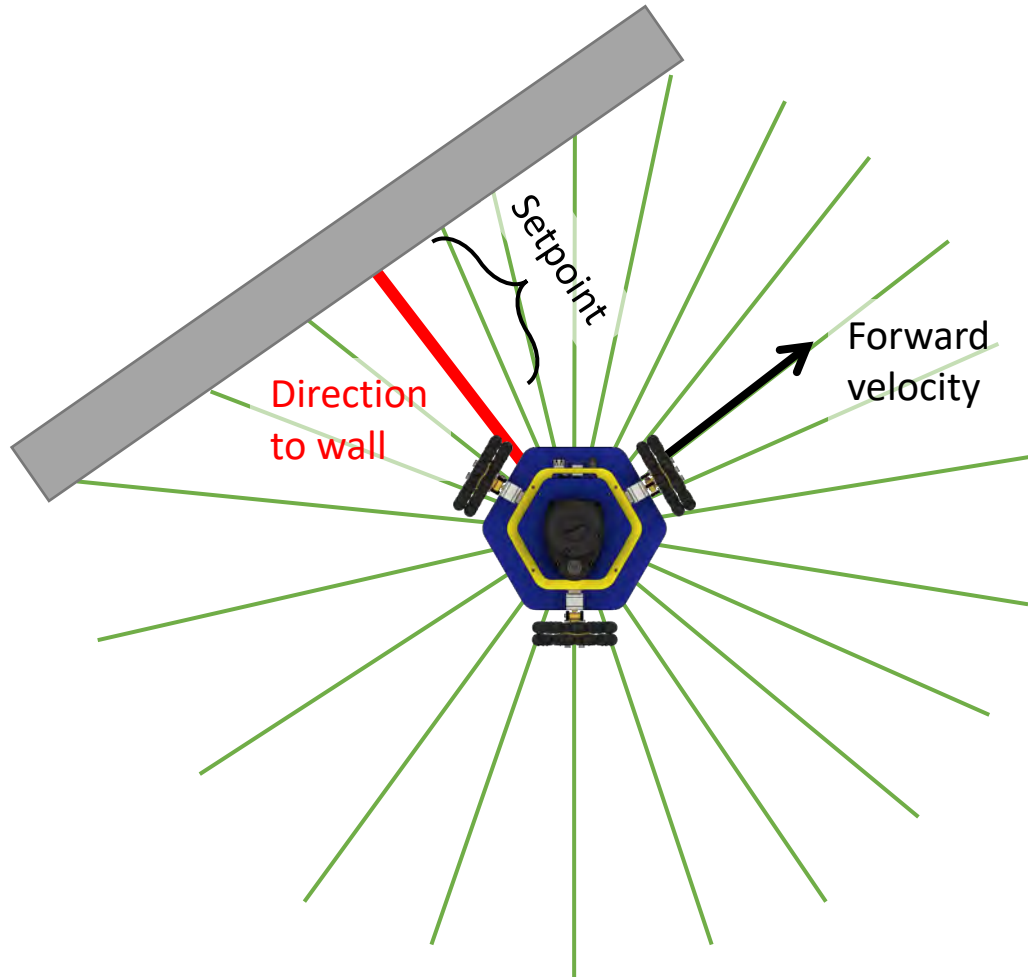
What is  $v_{to\_wall}$ ?

$$v_{to\_wall} = [1 \cdot \cos(\text{angle\_to\_wall}), 1 \cdot \sin(\text{angle\_to\_wall}), 0]$$

The diagram shows a 2D coordinate system with x and y axes. The x-axis is vertical and the y-axis is horizontal. A blue arrow labeled 'x' points downwards from the top of the x-axis. A blue arrow labeled 'z' points to the right from the right side of the equation. A blue arrow labeled 'y' points to the right from the right side of the equation.

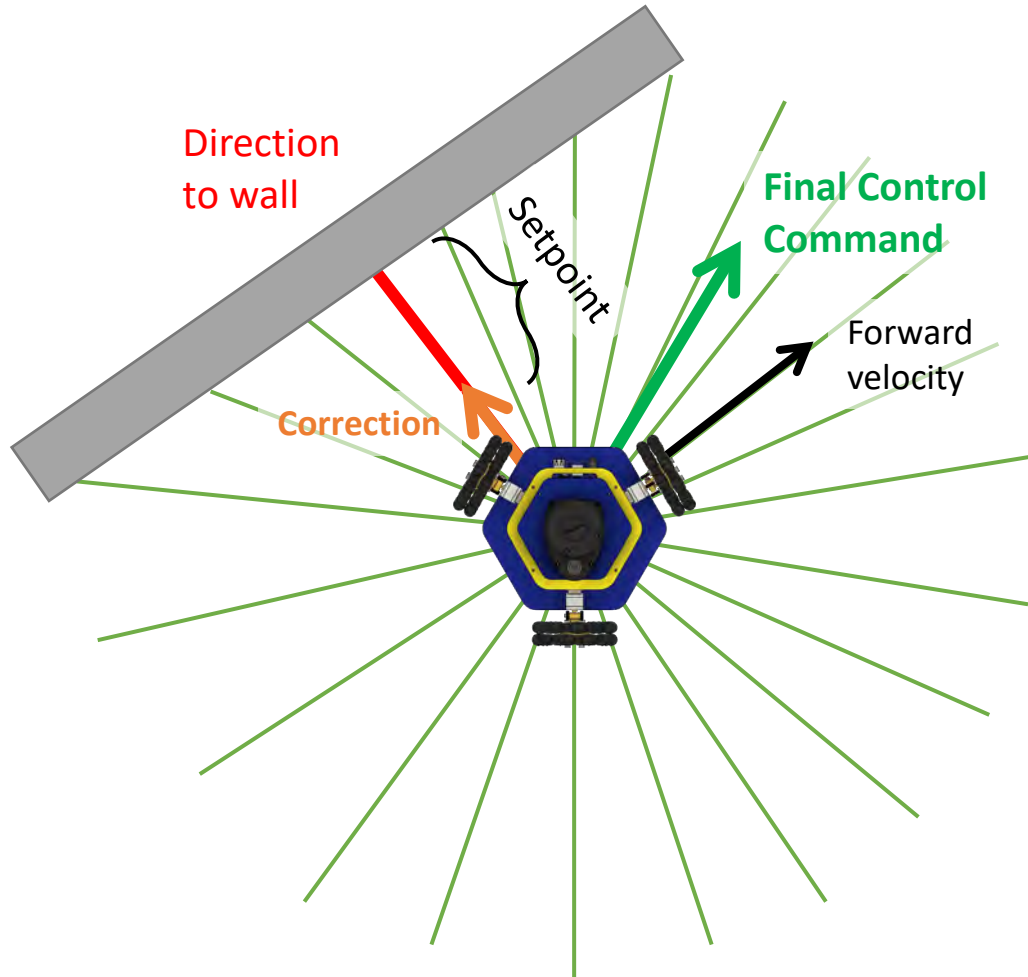
If we set the magnitude of  $v_{to\_wall}$  to 1,  $v_{forward}$  will have magnitude 1. We can multiply by a chosen drive velocity before sending the control commands to the robot.

# Wall Following: What are we missing?



*The robot should maintain a setpoint distance from the wall!*

# Wall Following: Bang-Bang Control

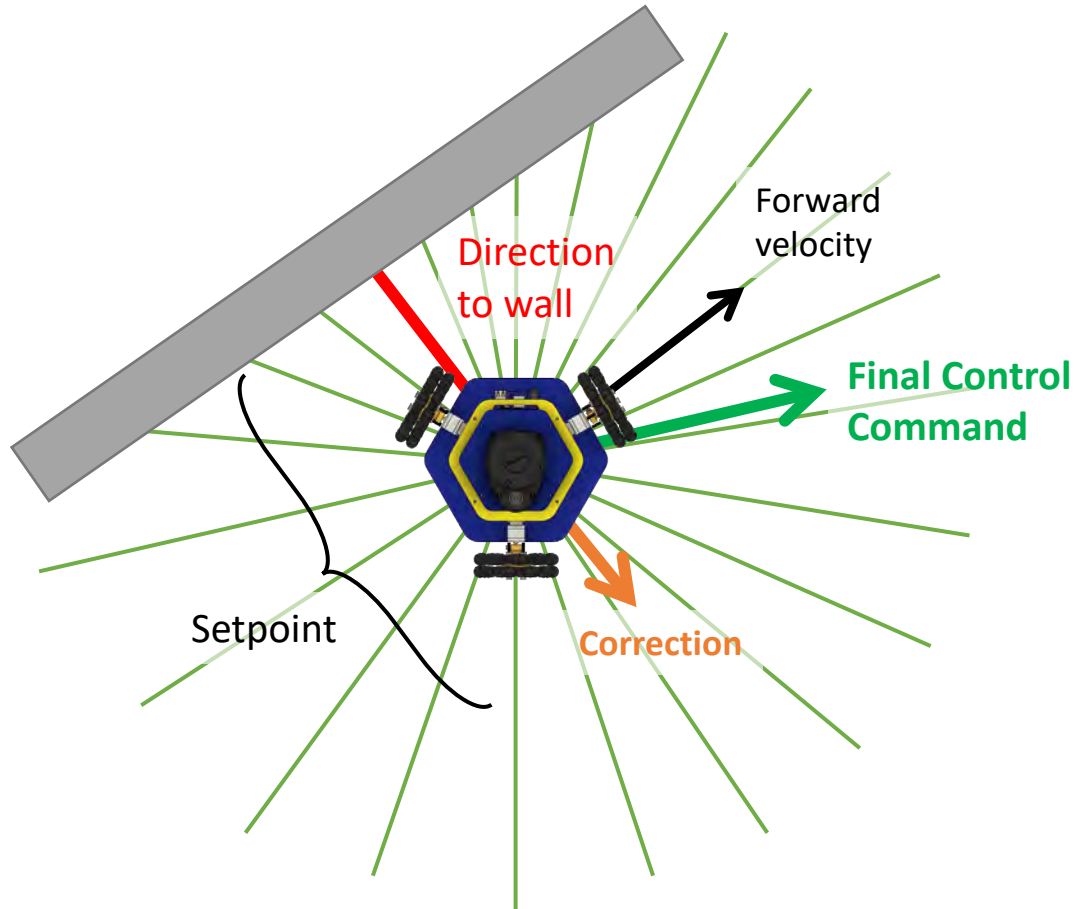


We can add a **correction** to the forward velocity to drive the robot closer or farther from the wall.

**Too far from wall! Move closer.**

The **final control command** is the forward velocity plus the correction (a vector addition!).

# Wall Following: Bang-Bang Control



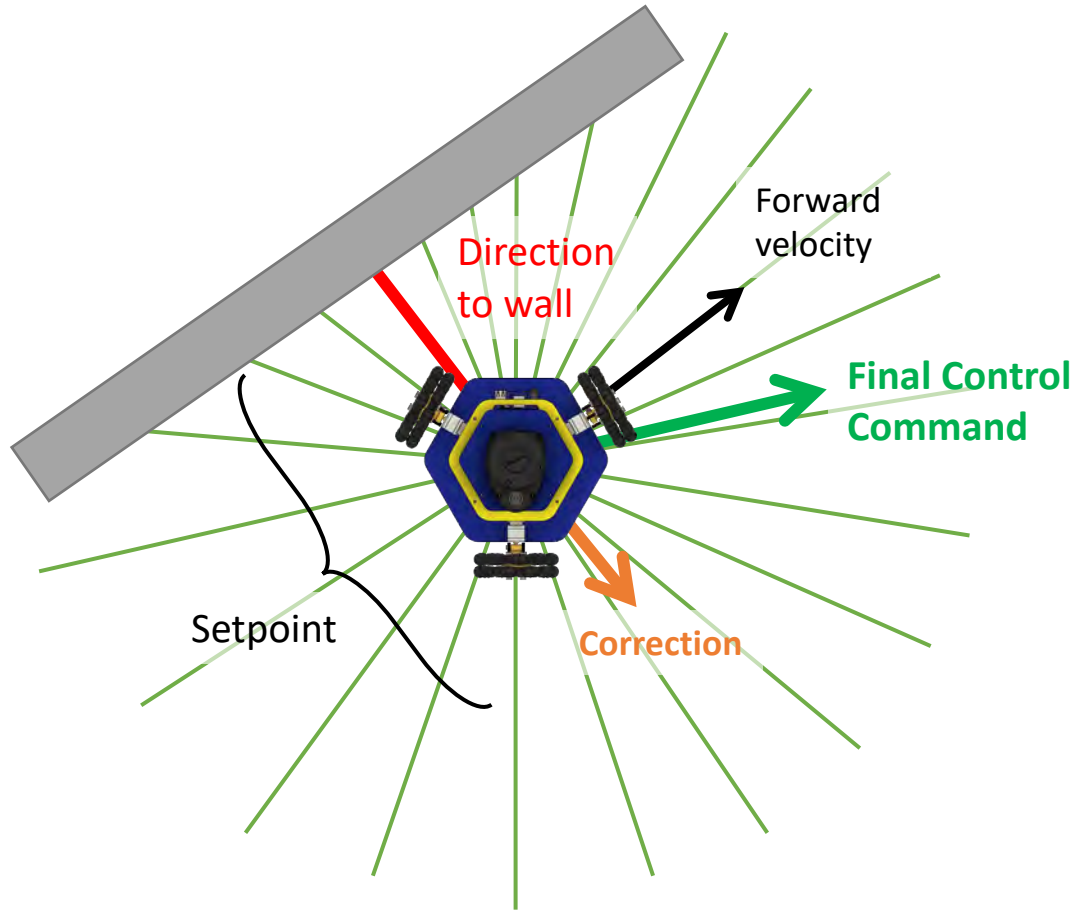
We can add a **correction** to the forward velocity to drive the robot closer or farther from the wall.

**Too close to wall! Move farther.**

For **Bang-Bang control**, the magnitude of the **correction** is *fixed*.

For **P-control**, the magnitude of the **correction** is *proportional to the error*.

# Wall Following: Algorithm



Loop forever:

1. Read a scan from the lidar
2. Find the shortest ray
3. Compute a vector pointing to the wall
4. Take a cross product to find the forward velocity
5. Compute the correction vector
6. Compute the final control command
7. Send the control command to the robot



# TODO: Today

1. Get `findMinDist()` function working
  - Used in both the 2D control in-class activity and P1.2
2. [*Optional*] Finish 2D control activity from Wednesday
3. Work on Project 1
  - i. P1.1 (Drive Square) should be finished and pushed to GitHub
  - ii. P1.2 (Drive Safe) can be finished once `findMinDist()` is working
  - iii. For P1.3 (Wall Following), start with the cross product and driving parallel to the wall. Then add the correction.